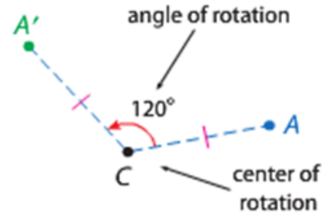
1 Draw Rotations

In Lesson 4-7, you learned that a rotation or *turn* moves every point of a preimage through a specified angle and direction about a fixed point.

KeyConcept Rotation

A rotation about a fixed point, called the **center of rotation**, through an angle of $x°$ is a function that maps a point to its image such that

- if the point is the center of rotation, then the image and preimage are the same point, or
- if the point is not the center of rotation, then the image and preimage are the same distance from the center of rotation and the measure of the **angle of rotation** formed by the preimage, center of rotation, and image points is x.



A′ is the image of *A* after a 120° rotation about point *C*.

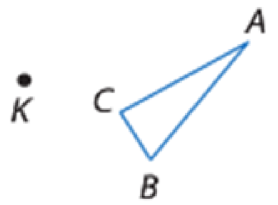
The direction of a rotation can be either clockwise or counterclockwise. Assume that all rotations are counterclockwise unless stated otherwise.

clockwise

counterclockwise

Example 1 Draw a Rotation

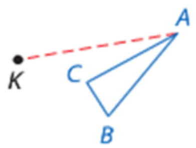
Copy $\triangle ABC$ and point K. Then use a protractor and ruler to draw a 140° rotation of $\triangle ABC$ about point K.



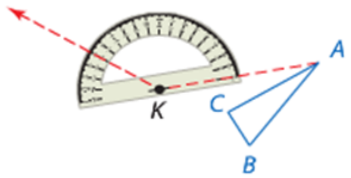
SKIP

SKIP

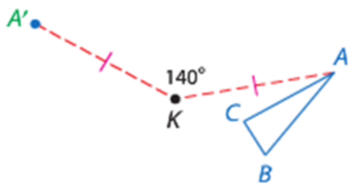
Step 1 Draw a segment from A to K.



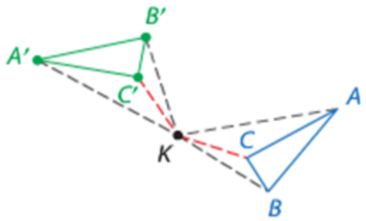
Step 2 Draw a $140°$ angle using $\overline{KA}$.



Step 3 Use a ruler to draw A' such that $KA' = KA$.

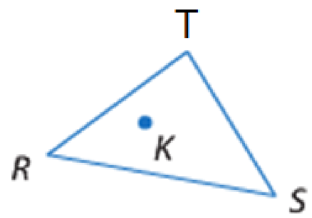


Step 4 Repeat Steps 1–3 for vertices B and C and draw $\triangle A'B'C'$.



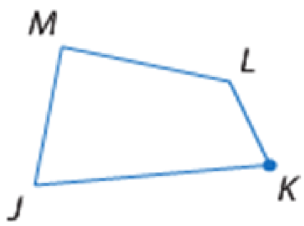
Copy each figure and point *K*. Then use a protractor and ruler to draw a rotation of the figure the given number of degrees about *K*.

1A. 65° SKIP



1B. 170°

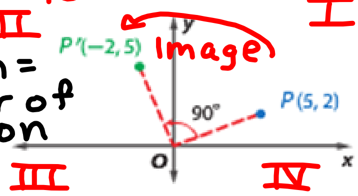
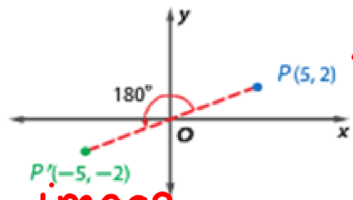
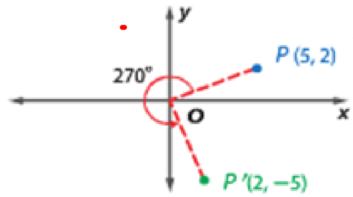
SKIP



2 Draw Rotations in the Coordinate Plane

When a point is rotated 90°, 180°, or 270° counterclockwise about the origin, you can use the following rules.

KeyConcept Rotations in the Coordinate Plane

90° Rotation To rotate a point 90° counterclockwise about the origin, multiply the y-coordinate by -1 and then interchange the x- and y-coordinates. **Symbols** $(x, y) \rightarrow (-y, x)$ Rule	**Example** 90° ccw origin = center of rotation 
180° Rotation To rotate a point 180° counterclockwise about the origin, multiply the x- and y-coordinates by -1. **Symbols** $(x, y) \rightarrow (-x, -y)$ * Γorigin *	**Example** 
270° Rotation To rotate a point 270° counterclockwise about the origin, multiply the x-coordinate by -1 and then interchange the x- and y-coordinates. **Symbols** $(x, y) \rightarrow (y, -x)$	**Example** 

StudyTip

Clockwise Rotation Clockwise rotation can be designated by a negative angle measure. For example a rotation of −90° about the origin is a rotation 90° clockwise about the origin.

StudyTip

360° Rotation A rotation of 360° about a point returns a figure to its original position. That is, the image under a 360° rotation is equal to the preimage.

R_{-90} = clockwise 90° = R_{360-90} = R_{270} ccw

R_{-180} = clockwise 180° = $R_{360-180}$ = R_{180} ccw

R_{-270} = clockwise 270° = $R_{360-270}$ = R_{90} ccw

Example 2 Rotations in the Coordinate Plane

Triangle PQR has vertices $P(1, 1)$, $Q(4, 5)$, and $R(5, 1)$. Graph $\triangle PQR$ and its image after a rotation 90° about the origin.

ccw

Multiply the y-coordinate of each vertex by −1 and interchange.

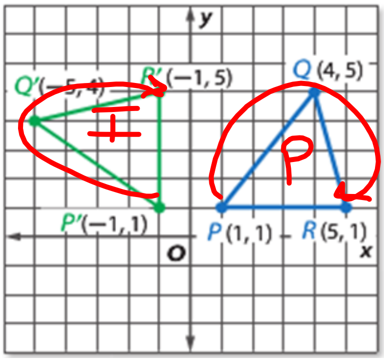
$(x, y) \rightarrow (-y, x)$ Rule

$P(1, 1) \rightarrow P'(-1, 1)$

$Q(4, 5) \rightarrow Q'(-5, 4)$

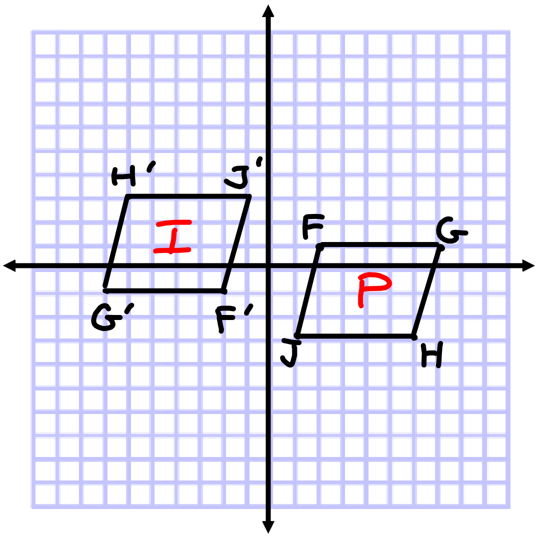
$R(5, 1) \rightarrow R'(-1, 5)$

Graph $\triangle PQR$ and its image $\triangle P'Q'R'$.



Direct Isometry

2. Parallelogram *FGHJ* has vertices *F*(2, 1), *G*(7, 1), *H*(6, −3), and *J*(1, −3). Graph *FGHJ* and its image after a rotation 180° about the origin.



R_{180} $(x,y) \rightarrow (-x,-y)$

$F(2,1) \rightarrow F'(-2,-1)$

$G(7,1) \rightarrow G'(-7,-1)$

$H(6,-3) \rightarrow H'(-6,3)$

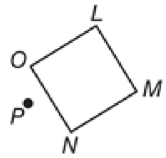
$J(1,-3) \rightarrow J'(-1,3)$

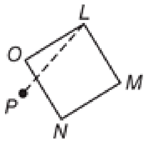
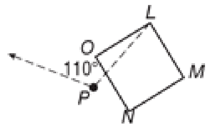
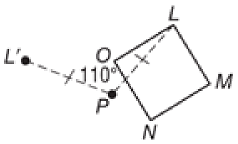
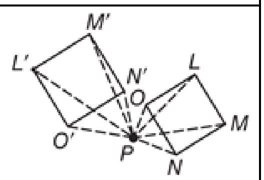
Draw Rotations A **rotation** is a transformation that moves every point of the preimage through a specified angle, $x°$, and direction about a fixed point called the **center of rotation**.

- If the point being rotated is the center of rotation, then the image and preimage are the same point.
- If the point being rotated is not the center of rotation, then the image and preimage are the same distance from the center of rotation and the measure of the angle of rotation formed by the preimage, center of rotation, and image points is x.

Example: Use a protractor and ruler to draw a 110° rotation of square *LMNO* about point *P*.

SKIP

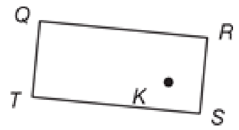


Step 1 Draw a segment from vertex L to point P. 	**Step 2** Draw a 110° angle using $\overline{PL}$ as one side. 
Step 3 Use a ruler to draw L' such that $PL' = PL$. 	**Step 4** Repeat steps 1–3 for vertices M, N, and O and draw square $L'M'N'O'$. 

Exercises SKIP

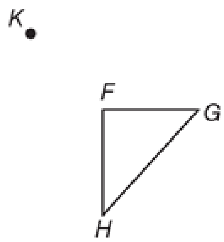
Use a protractor and a ruler to draw the specified rotation of each figure about point *K*.

1. 75°



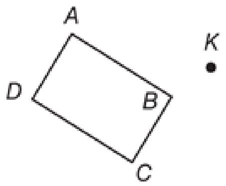
2. 45°

SKIP



3. 135°

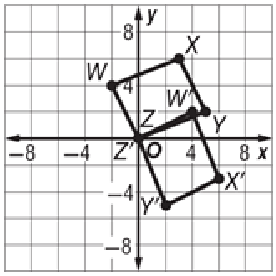
SKIP



Draw Rotations In The Coordinate Plane The following rules can be used to rotate a point 90°, 180°, or 270° counterclockwise about the origin in the coordinate plane.

To rotate	Procedure
90°	Multiply the y-coordinate by −1 and then interchange the x- and y-coordinates. $(-y, x)$
180°	Multiply the x- and y-coordinates by −1. $(-x, -y)$
270°	Multiply the x-coordinate by −1 and then interchange the x-and y-coordinates. $(y, -x)$

Example: Parallelogram $WXYZ$ has vertices $W(-2, 4)$, $X(3, 6)$, $Y(5, 2)$, and $Z(0, 0)$. Graph parallelogram $WXYZ$ and its image after a rotation of 270° about the origin.



Multiply the x-coordinate by −1 and then interchange the x-and y-coordinates.

$(x, y) \rightarrow (y, -x)$

$W(-2, 4) \rightarrow W'(4, 2)$

$X(3, 6) \rightarrow X'(6, -3)$

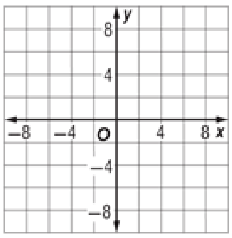
$Y(5, 2) \rightarrow Y'(2, -5)$

$Z(0, 0) \rightarrow Z'(0, 0)$ → (0,0) is the center of rotation.

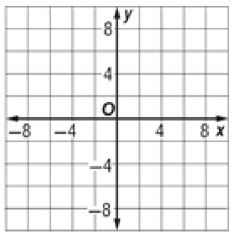
Exercises

Graph each figure and its image after the specified rotation about the origin.

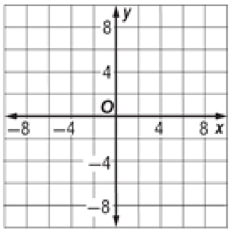
1. trapezoid *FGHI* has vertices *F*(7, 7), *G*(9, 2), *H*(3, 2), and *I*(5, 7); 90°



2. △*LMN* has vertices *L*(−1, −1), *M*(0, −4), and *N*(−6, −2); 90°



3. $\triangle ABC$ has vertices $A(-3, 5)$, $B(0, 2)$, and $C(-5, 1)$; 180°



4. parallelogram $PQRS$ has vertices $P(4, 7)$, $Q(6, 6)$, $R(3, -2)$, and $S(1, -1)$; 270°